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DISTORTION IN X-BAND DETECTORS

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DISTORTION IN X-BAND DETECTORS

Abstract

Measurements of distortion in balanced and unbalanced X-band mixers indicate that the balanced mixer reduces second harmonic distortion by at least 10 db. Third harmonic distortion is not reduced. An approximate theory of balanced mixer conversion is sketched.

M.W.P. Strandberg

Approved by: A. E. Whitford
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Title page
6 numbered pages
12 pages of figures

DISTORTION IN X-BAND DETECTORS

Introduction:

There are several instances when it is desired to know the fidelity of the output of an X-band mixer operated at high signal level. One example is an automatic frequency control (a.f.c.) design. In this case both signal power and local oscillator power are in the milliwatt region, and it is imperative to evaluate the distortion in the beat frequency currents that will give rise to misleading a.f.c. information. There is reason, as explained later, to believe this distortion is minimized in a balanced mixer, so measurements were made with an X-band balanced mixer over a wide range of crystal operating power.

Method of Measurement:

There are a number of methods of taking the necessary data. The obvious method would be to set up a mixer with variable attenuators on both the transmitter and the local oscillator (L.O.) inputs. These attenuators could be varied as the i-f output voltage is measured for the fundamental, second and higher harmonics of the beat product of the L.O. and transmitter. The arrangement is inconvenient for a number of reasons.

First, the high power level of the pulsed transmitter requires care in attenuator calibrations, and the absence of any leakage around the attenuators. And second, the pulsed r-f signal, detected in the crystal has video side-bands which must be taken into account in any measurement of actual mixer power. It might seem that this is the operating condition in a radar set anyway, so this video power at the i-f added to the actual i-f will give a more realistic reading. This is not the case, or not practical to do, at least, for the spectrum of this video signal will vary violently with transmitter pulse shape and hence this factor will be a function of the receiver i-f and the transmitter pulse shape. And third, in measuring harmonic content which is very small compared to the fundamental care must be taken to correct for the skirt side bands of the fundamental. For these reasons it was decided to make the measurements in another way.

Two c.w., r-f carriers were fed into the mixer through variable attenuators and the value of the c.w. fundamental and harmonic output was measured. This method allowed the signal power to be measured directly with a thermistor bridge. It also avoided the need to separate the i-f and the video pulse hash in the measurements. The apparatus was extremely simple.

Figure 1 shows a schematic of the apparatus. The balanced mixer fed a single i-f stage of variable gain. This stage, a single tuned circuit, drove a grid leak detector and the output was measured with a bridge meter on the detector tube plate. Changing the gain varied the bandwidth of the i-f amplifier, but this caused no harm since this stage was fed with a 10 mc/s wide double tuned circuit which greatly reduced the i-f off-tune, or skirt, response. Also the lowest gain and consequently the broadest bandwidth was used with the the largest input signal; and the smaller signals were measured with a smaller bandwidth and higher gains. Consequently negligible extraneous skirt power from the fundamental was measured. The over-all gain was measured with a GR804 signal generator at 30 mc/s to be 31 db with lowest gain, and the actual i-f crystal output was measured at high level with a CV58 diode detector and the overall amplification checked to within .. db.

One signal source was an X-band klystron, the 419B. The other was a 2K25 oscillator stabilized in the microwave discriminator circuit. (R.L. No. 662 Microwave Frequency Discriminator) The 419B is very stable in frequency, but it is difficult to tune and still keep the power output constant. So it was kept fixed at 9000 mc/s. The 2K25 was tuned with the discriminator cavity which controls the reflector voltage and the power kept maximized by adjusting the rhumbatron cavity as the tube was tuned. By reading the frequency to which the cavity was tuned at each measurement the order of any harmonic was easily calculated.

As a check and reference to evaluate the improvement in fidelity gained with a balanced converter some measurements were made with a single ended input. One mixer crystal was grounded and the receiver lead for that crystal was terminated in a 400 ohm resistor to maintain the loading on the receiver input. The curves of data taken from these readings are included in this report as a comparative check.

Discussion of Results:

This data is best taken as purely empirical; however, an over-simplified theory does give some insight into the mechanism of the behavior of the distortion.

A simple method of solution of this problem was suggested by H. C. Torrey of Radiation Laboratory. It can be shown that the short circuit current of a crystal can be represented by the equations:

$$i_{sc} = A(e^{\alpha V} - 1)$$

In this equation A is a scaling factor in amperes, V is the voltage across the crystal barrier layer in volts and α is about 10 volts^{-1}. In this approximation V can be taken as the input signal voltage, although this is not strictly true since actually

$$V = V_{sig} - i_{sc}R$$

where R is the spreading resistance in series with the barrier layer. Even so, the equation does not hold above $i \sim 10$ milliampere, for no account is taken of saturation phenomena in the crystal.

With these restrictions it can be shown (Appendix 1) with a voltage input of the form

$$V = V_1 \cos \omega_1 t + V_2 \cos (\omega_1 + \beta) t$$

that the coefficients of various harmonics of β are

$$\begin{aligned} i_{sc} = \Big\{ & 2I_1 (\alpha V_1)\, I_1 (\alpha V_2) \cos \beta t \\ & + 2I_2 (\alpha V_1)\, I_2 (\alpha V_2) \cos 2\beta t \; \ldots\ldots \\ & + 2I_n (\alpha V_1)\, I_n (\alpha V_2) \cos n\beta t \\ & + C + \text{terms in } m\omega_1,\ n(\omega_1 + \beta),\ n(\omega_1 + \beta)\, t \pm m(\omega_1 t) \Big\} A \end{aligned}$$

$n \neq m$ for lower sign

$I_n\ (av)$ is the modified Bessel function, C is a constant, and n and m are integers greater than zero.

The function of interest is

$$\frac{\text{Fundamental current}}{\text{Harmonic current}} = \frac{I_1\ (aV_1)\ I_1\ (aV_2)}{I_n\ (aV_1)\ I_n\ (aV_2)}$$

Here again an assumption must be made, for this function is in terms of receiver output signal only if the available power varies as the square of this quantity, or, in other words, if the source i-f impedance is constant for all of these currents. Lacking any better approximation it has been assumed that the i-f impedance does not vary for these harmonic quanitities. Actually this is a poor approximation, for the source impedance for the harmonics is probably controlled by the impedance for the fundamental since this fundamental comprises the greatest portion of the beat currents flowing. So harmonic impednnces would be close to that of the fundamental when the fundnmental curreots are working into a fixed load. However, the measurements are made with the fundamental working into some load when it is measured, and into a near short when the harmonics are measured. This should give a change in i-f impedance. However, since no better approximation is available the function of fundamental to harmonic currents has been plotted as a first order approximation to the actunl nvailable powers. These curves for 2nd and 3rd harmonics are shown in figures 2 and 3.

These curves may be compared with experimental dnta taken for harmonic output of a single ended mixer. These data are plotted in figures 4 and 5.

In comparing the "theoretical" and experimental curves it will be noted thnt there is slight agreement in absolute value. The second harmonic curves, for instance, show a decrease in relative harmonic content due to saturation, a factor not taken into account in the calculation of curves of figure 2. However, the second harmonic curves show agreement in the slope of the curves, and in general shape.

The curves for the third harmonic figures 3 and 5, again show poor correlation in absolute value, and errors due to saturation of the crystal. However, the general slopes and shape show agreement. It should be noted that the experimental data with low signal input can contain large errors above about 40 db.

From theoretical considerntions(Appendix II) it can be shown that with balanced generators, a double ended mixer should have no even harmonic content. In practice crystals are not exnctly paired in chnracteristics so one would expect the even-harmonic content, being the difference between two varinble numbers, to vary over a wide range from crystal pair to crystal pair. However, the odd harmonics, being additive, should show small variation. This is in fact the case experimentally. Figures 6, 7, 8, show typical curves for the second harmonics generated in a"balanced" mixer. Figure 9, shows the third harmonic generated, and is the average of four sets of crystals. It can be seen that the second harmonic curves show too great variation to be so plotted.

Figures 6 and 7 show that in general, with ordinary crystals the third harmonic content may be considered as the "noise" level in an a.f.c. circuit. However, figure 8 shows that with poorly matched crystals one must be prepared to deal with up to 10 db more second harmonic. In the ordinary case, however, one may expect to diminish the second harmonic content in going from a single to a double ended mixer by about 10 db.

The third harmonic content shown in figure 9 compares favorably with that of the single ended mixer in figure 5.

It is interesting to note that in the course of the measurements on the double-ended mixer no 4th harmonics were observed, but 5th harmonics were.

Since the crystal output is limited by saturation in the crystal, one would expect 3 db more output at high level from the double ended mixer. This is the case as figures 10 and 11 show. Figure 10 shows the output voltage of a single ended mixer working into a given load, and figure 11 shows the output of a double ended mixer working into the same load. Absolute calibration is difficult to do with this type of apparatus, so not too much confidence should be given to these curves. They are reproducible from crystal to crystal, but their magnitude is probably correct to only ± 1.5 db.

As a comparison of single and double ended mixers figure 12 has been plotted. This shows 2nd and 3rd harmonics for the case of 1 ma. L.O., input with variable signal input.

The composite graphs in this report, figures 9 and 11 are composed of data from three sets of crystals. Figures 4, 5 and 10, curves showing single ended operation, data are constructed from data taken on one crystal with random point checks with other crystals.

Curves 4 and 5 and 8 and 9 were used in figure 12.

Random sampling of points was also made as a rough check of the accuracy of the general trend.

Considering the gain and impedance tolerances allowed in the 1N23B by JAN specifications the point scatter is well within reason. Only figure 12 has been printed with the actual data showing in order to give an idea of this scattering. All other curves are smoothed curves. This has been done since the importance of this work is to ascertain tendenciesrather than individual crystal characteristics.

Summary:

With a single ended mixer one may expect the second harmonic of the input frequency in the crystal to be about 20 db below the fundamental and the third harmonic to be about 30 db below the fundamental.

With a double ended mixer one may expect 3 db more signal than with a single ended mixer. The second harmonic in this case will generally be below the third harmonic. So the "noise" level of an a.f.c. circuit is set by the third harmonic and is 30 db below the fundamental.

M. W. P. Strandberg
August 15, 1945

Appendix I

The writer is indebted to H. C. Torrey and R. H. Dicke for suggestions and discussion in this exposition.

One may assume a crystal law of the following form:

$$i_{sc} = A\left(e^{\alpha V} - 1\right)$$

where V is the barrier layer voltage, and α and A are constants. Neglecting the spreading resistance through which this current flows this equation then becomes:

$$i_{sc} = A\left(e^{\alpha V_1 \cos \omega_1 t}\, e^{\alpha V_2 \cos (\omega_1 + \beta)\, t} - 1\right)$$

with two sinusoidal input voltages of frequency difference β. The series form for these exponentials may be found by expanding in a Fourier series, and the series turn out to be:

$$i_{sc} = A\left\{\left[I_o(\alpha V_1) + 2\sum_{n=1} I_n(\alpha V_1)\cos n\,\omega_1 t\right]\left[I_o(\alpha V_2) + 2\sum_{m=1} I_m(\alpha V_2)\cos m\,(\omega_1 + \beta)\, t\right] - 1\right\}$$

(S. A. Schelkunoff, Electro-magnetic Waves, p. 55)

It can be seen that this reduces to the form

$$i_{sc} = A\left\{2I_1(\alpha V_1)\, I_1(\alpha V_2)\cos\beta t + \ldots 2I_n(\alpha V_1)\, I_n(\alpha V_2)\cos n\,\beta t\right.$$

$$\left. + \text{ constant} + \text{terms in } n\omega_1 t,\ m(\omega_1+\beta)t,\ n\omega_1 t \pm m(\omega_1 + \beta)t\right\}$$

Appendix II

In the case of a balanced converter one of the input voltages has a 180° phase shift at one crystal. Thus the short circuit currents in each detector are as follows:

$$i_{sc1} = A\left(e^{+\alpha V_1 \cos\omega_1 t}\, e^{\alpha V_2 \cos(\omega_1 + \beta)t} - 1\right)$$

$$i_{sc2} = A\left(e^{-\alpha V_1 \cos\omega_1 t}\, e^{\alpha V_1 \cos(\omega_1 + \beta)t} - 1\right)$$

The expansion of this last equation is then given as:

$$i_{sc2} = A\left\{\left[I_o(\alpha V_1) + 2\sum_{n=1}(-1)^n I_n(\alpha V_1)\cos\omega_1 t)\right]\right.$$

$$\left.\left[I_o(\alpha V_2) + 2\sum_{m=1} I_m(\alpha V_2)\cos m\,(\omega_1 + \beta)t(-1)\right]\right\}$$

In this test apparatus one crystal is operated to give positive output and the other a negative output. The two currents are then added algebraically in the output. To signify a negative detecting crystal the sign of i_{sc2} is changed so the sum is:

$$i_{sc\ input} = i_{sc1} + (-i_{sc2})$$

Using the equation developed in Appendix I this reduces to

$$i_{short\ circuit\ input} = A \Big\{ 2 \sum_{n=1} I_n\ (\alpha V_1)\ \cos n\omega_1 t \sum_{m=1} I_m(\alpha V_2)\ \cos\ (\omega_1 + \beta)t$$

$$-2 \sum_{n=1} (-1)^n\ I_n\ (\alpha V_1)\ \cos n\omega_1 t \sum_{m=1} I_m(\alpha V_2)\ \cos(\omega_1 + \beta)t + \text{non-cross product terms} \Big\}$$

It can be seen that all terms in even n will cancel out so this reduces to

$$i_{sc}\ \text{input} = A \Big\{ 4\ I_1(\alpha V_1) I_1(\alpha V_2)\ \cos\beta t + 4\ I_3(\alpha V_1)\ I_3\ (\alpha V_2)\ \cos 3\beta t + \cdots 4\ I_{2n-1}(\alpha V_1)\ I_{2n-1}(\alpha V_2)\ \cos(2n-1)\beta t + \text{other terms not integers of } \beta \Big\}$$

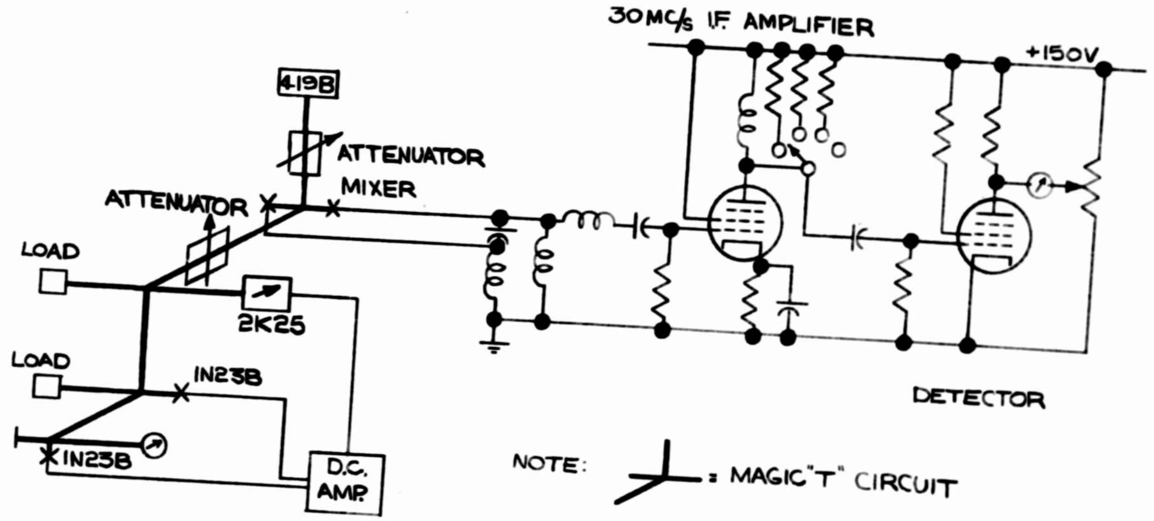
30MC/S I.F. AMPLIFIER
+150V
419B
ATTENUATOR
MIXER
ATTENUATOR
LOAD
2K25
LOAD
IN23B
IN23B
D.C. AMP.
DETECTOR
NOTE:
= MAGIC "T" CIRCUIT

FIGURE 1

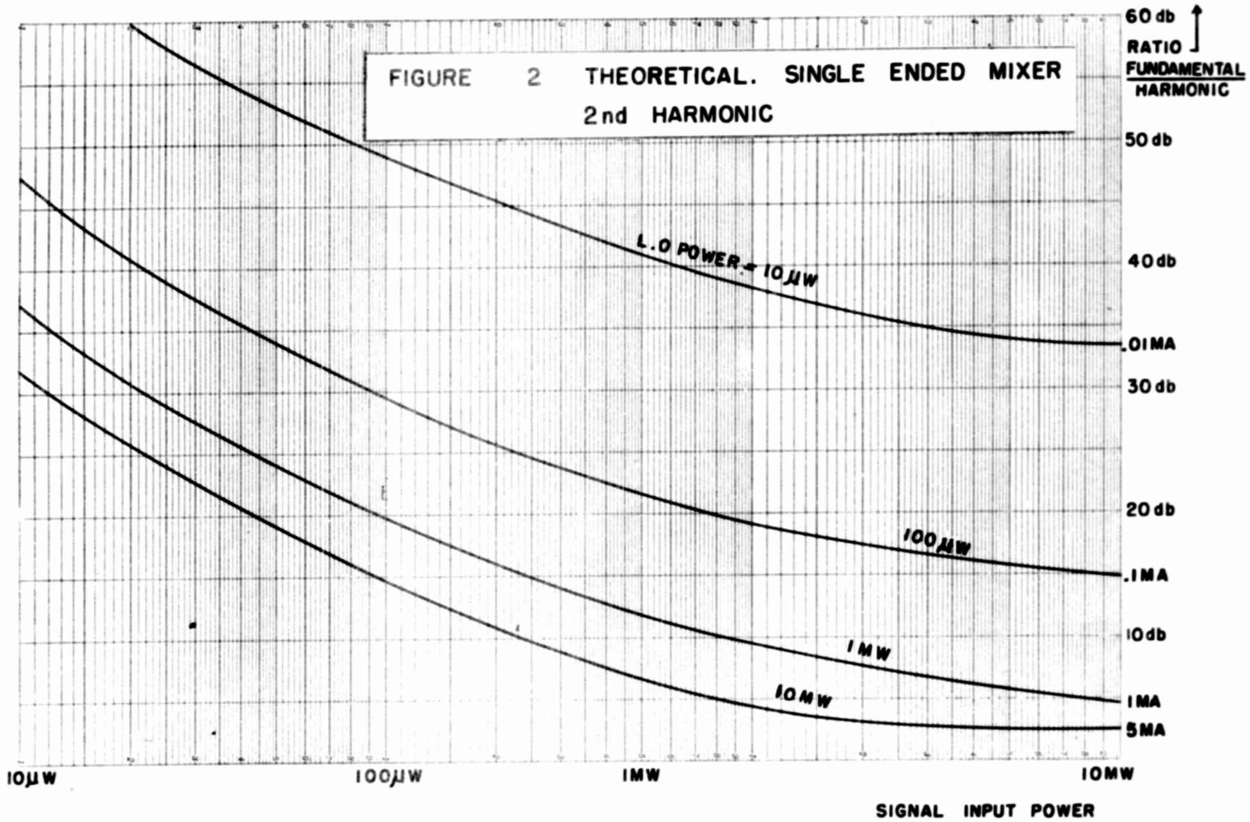

FIGURE 2 THEORETICAL. SINGLE ENDED MIXER
2nd HARMONIC

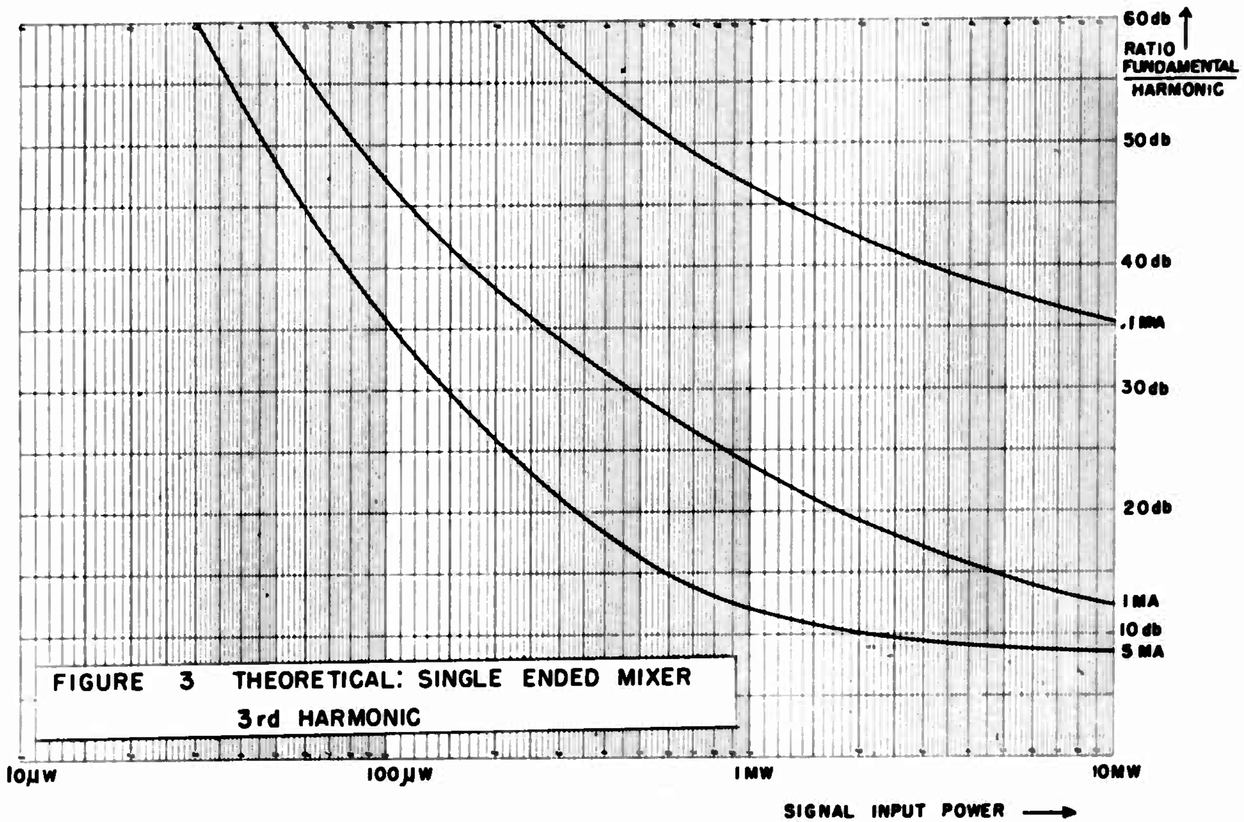

FIGURE 3 THEORETICAL: SINGLE ENDED MIXER
3rd HARMONIC

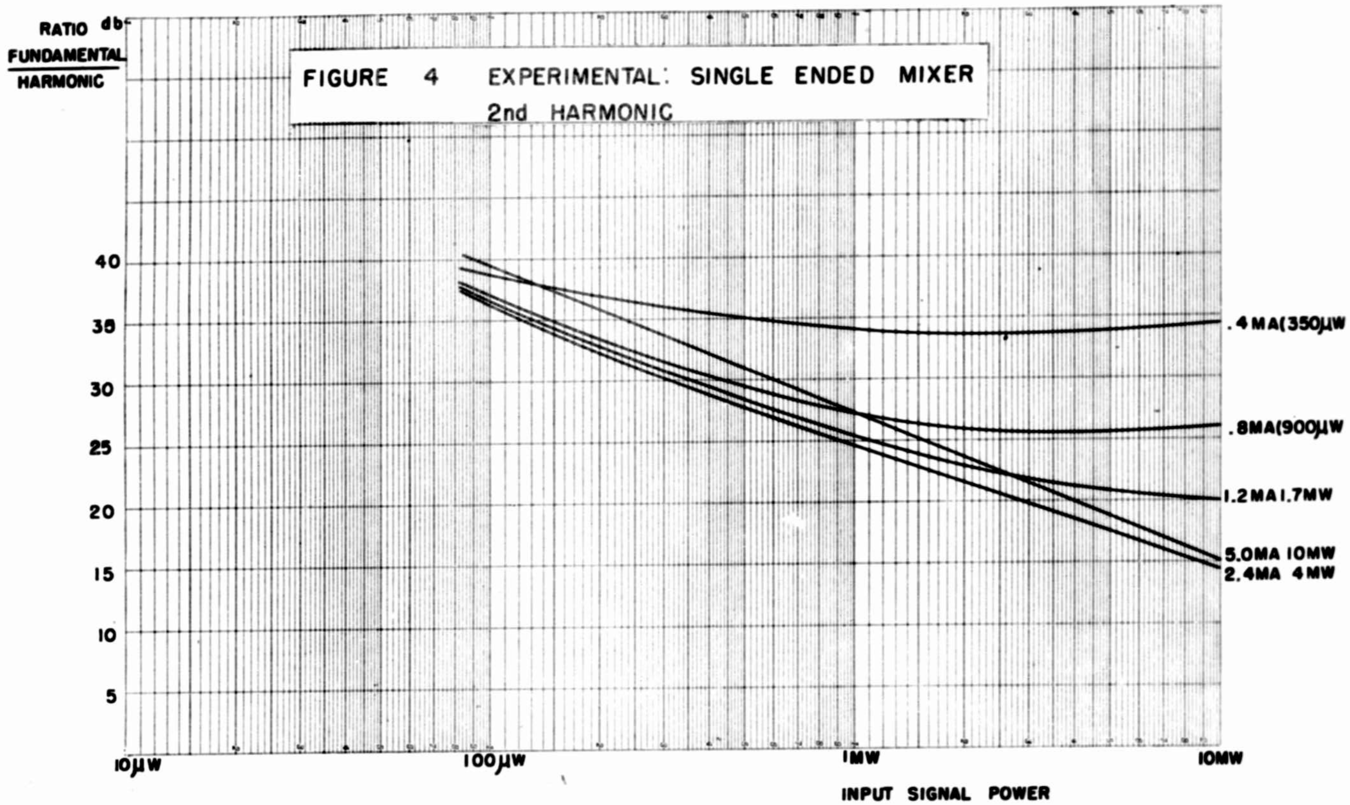

FIGURE 4 EXPERIMENTAL: SINGLE ENDED MIXER
2nd HARMONIC

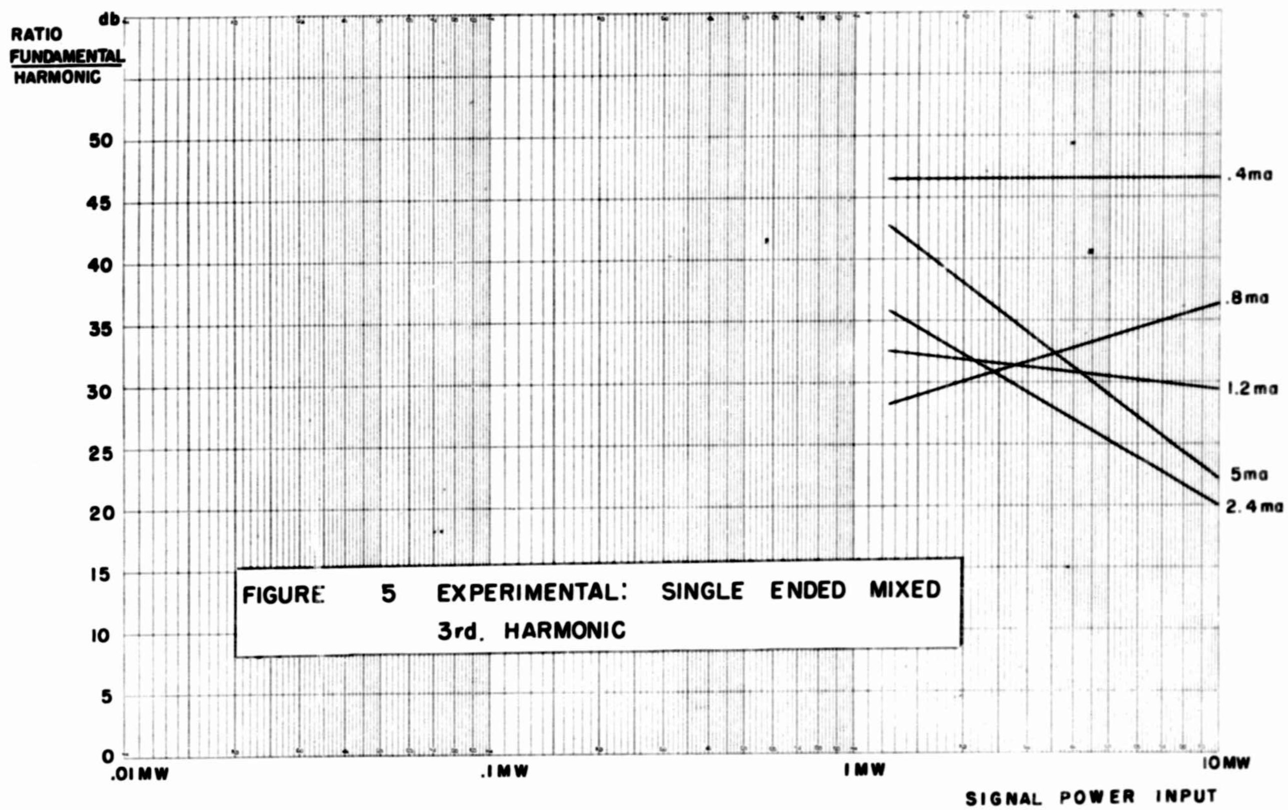

FIGURE 5 EXPERIMENTAL: SINGLE ENDED MIXED
3rd. HARMONIC

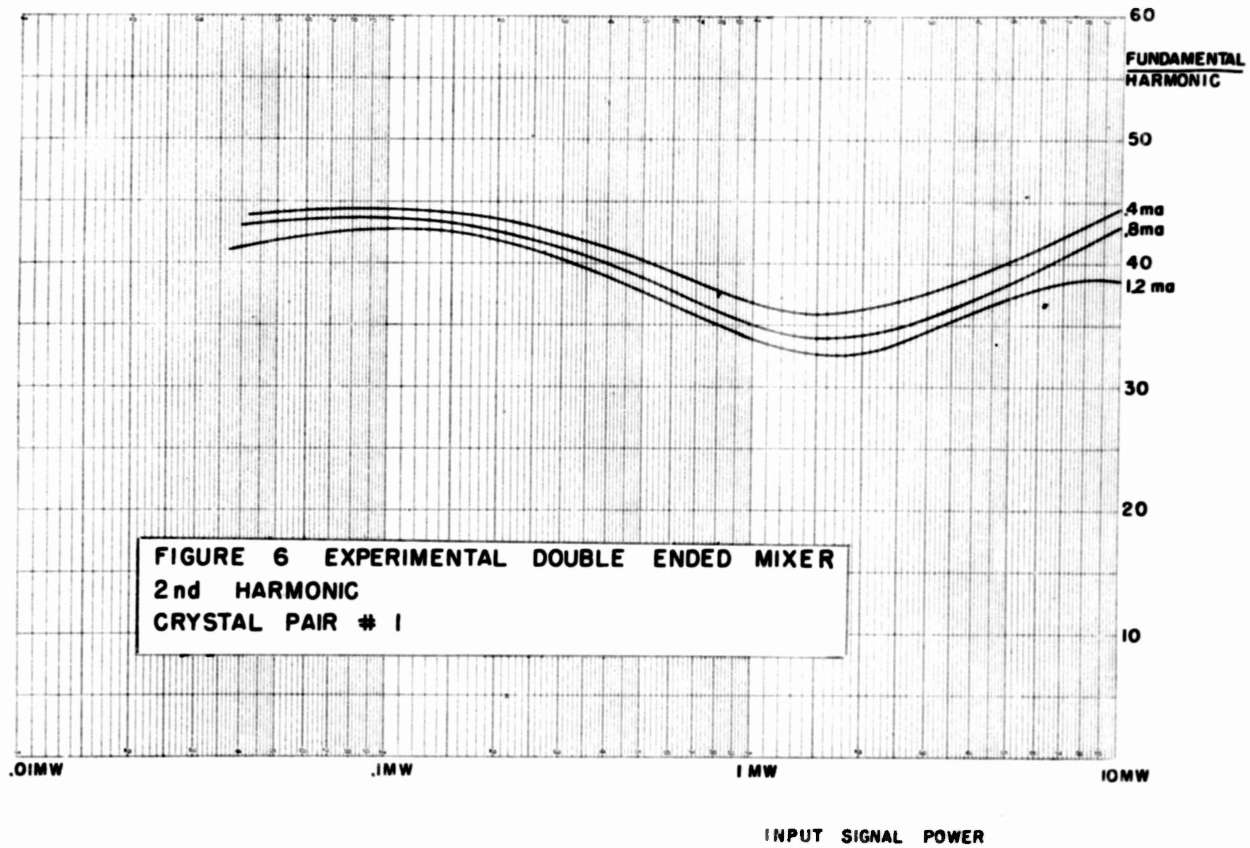

FIGURE 6 EXPERIMENTAL DOUBLE ENDED MIXER
2nd HARMONIC
CRYSTAL PAIR # 1

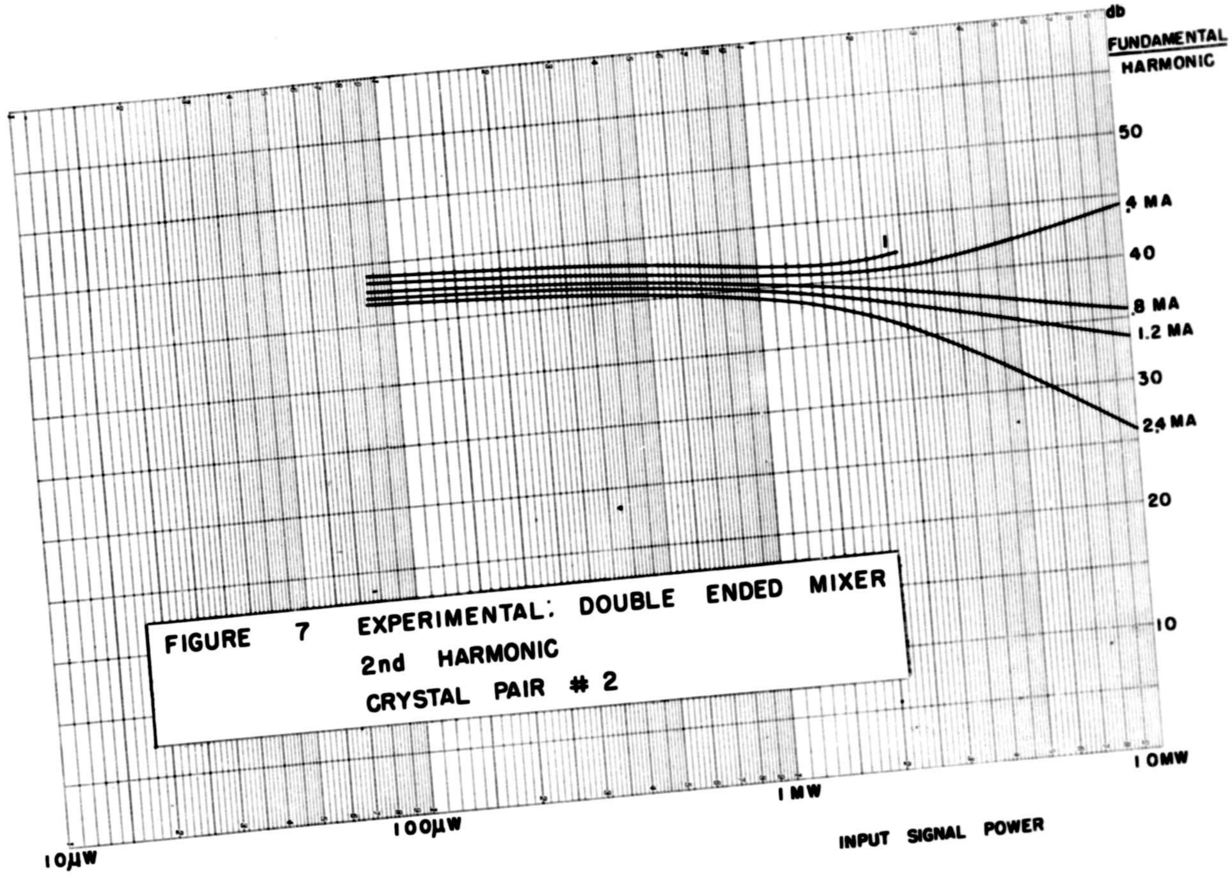

FIGURE 7 EXPERIMENTAL: DOUBLE ENDED MIXER
2nd HARMONIC
CRYSTAL PAIR # 2

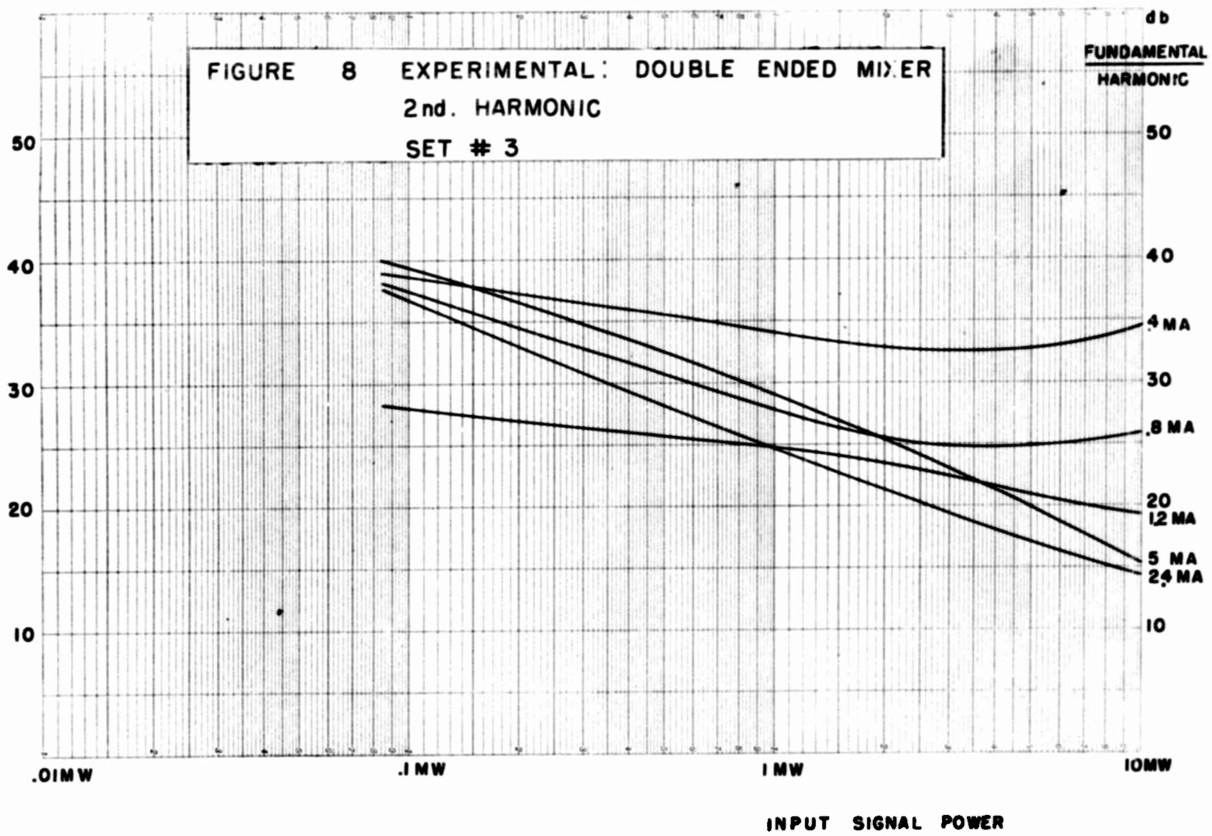

FIGURE 8 EXPERIMENTAL: DOUBLE ENDED MIXER
2nd. HARMONIC
SET # 3

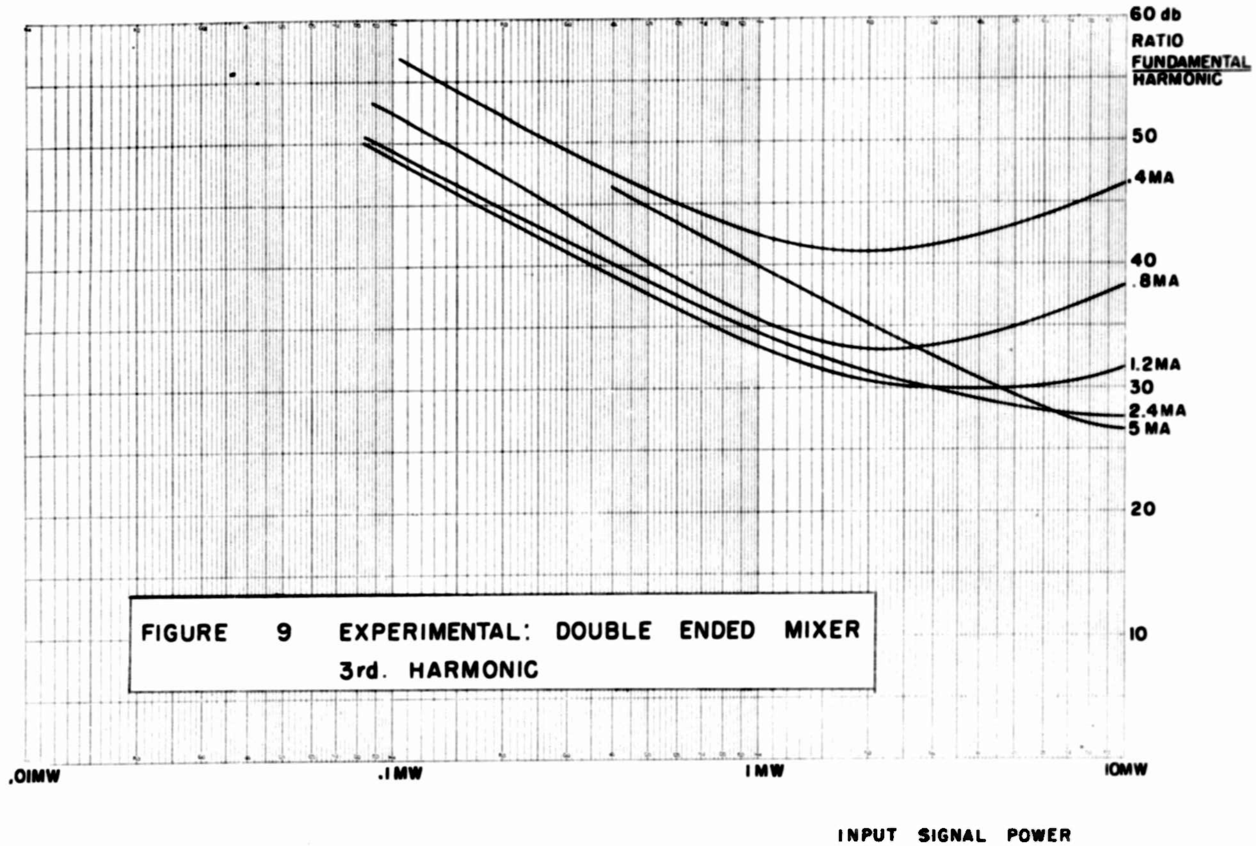

FIGURE 9 EXPERIMENTAL: DOUBLE ENDED MIXER
3rd. HARMONIC

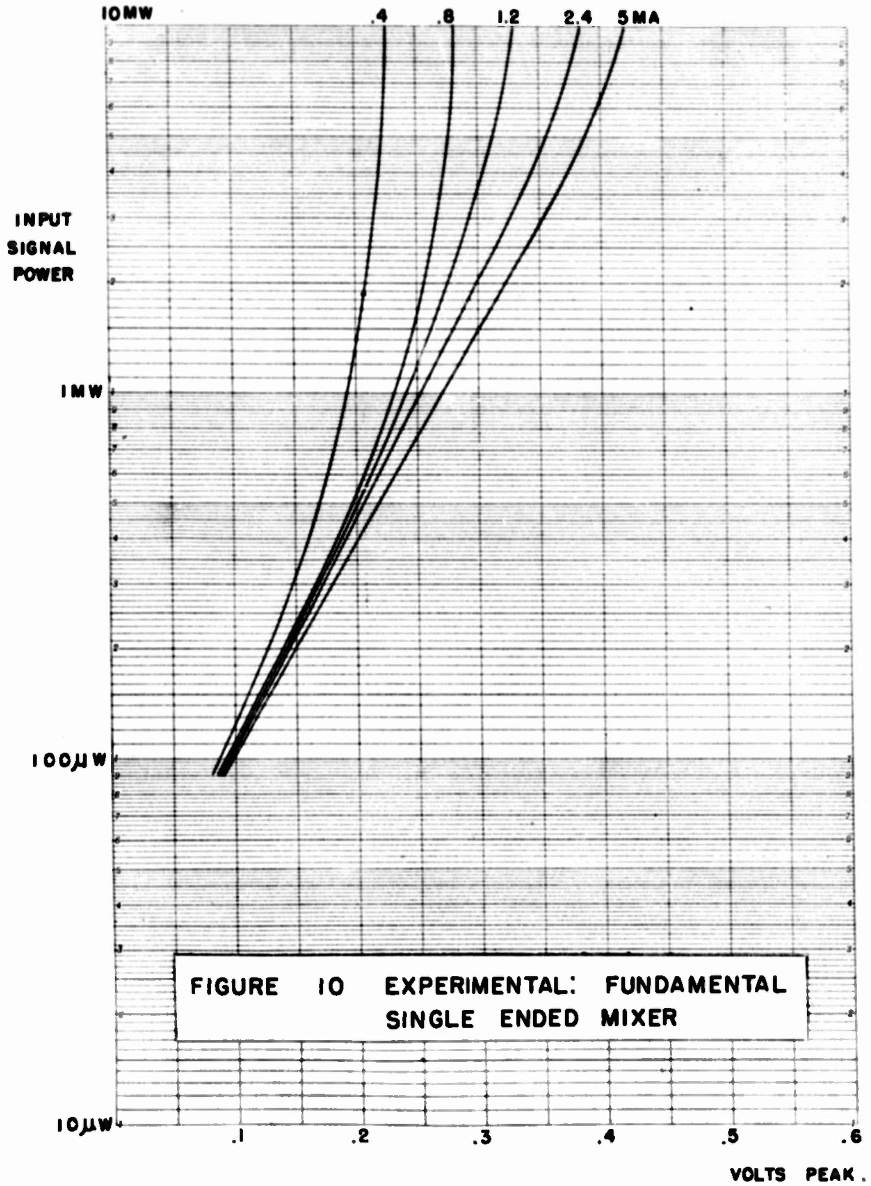

FIGURE 10 EXPERIMENTAL: FUNDAMENTAL SINGLE ENDED MIXER

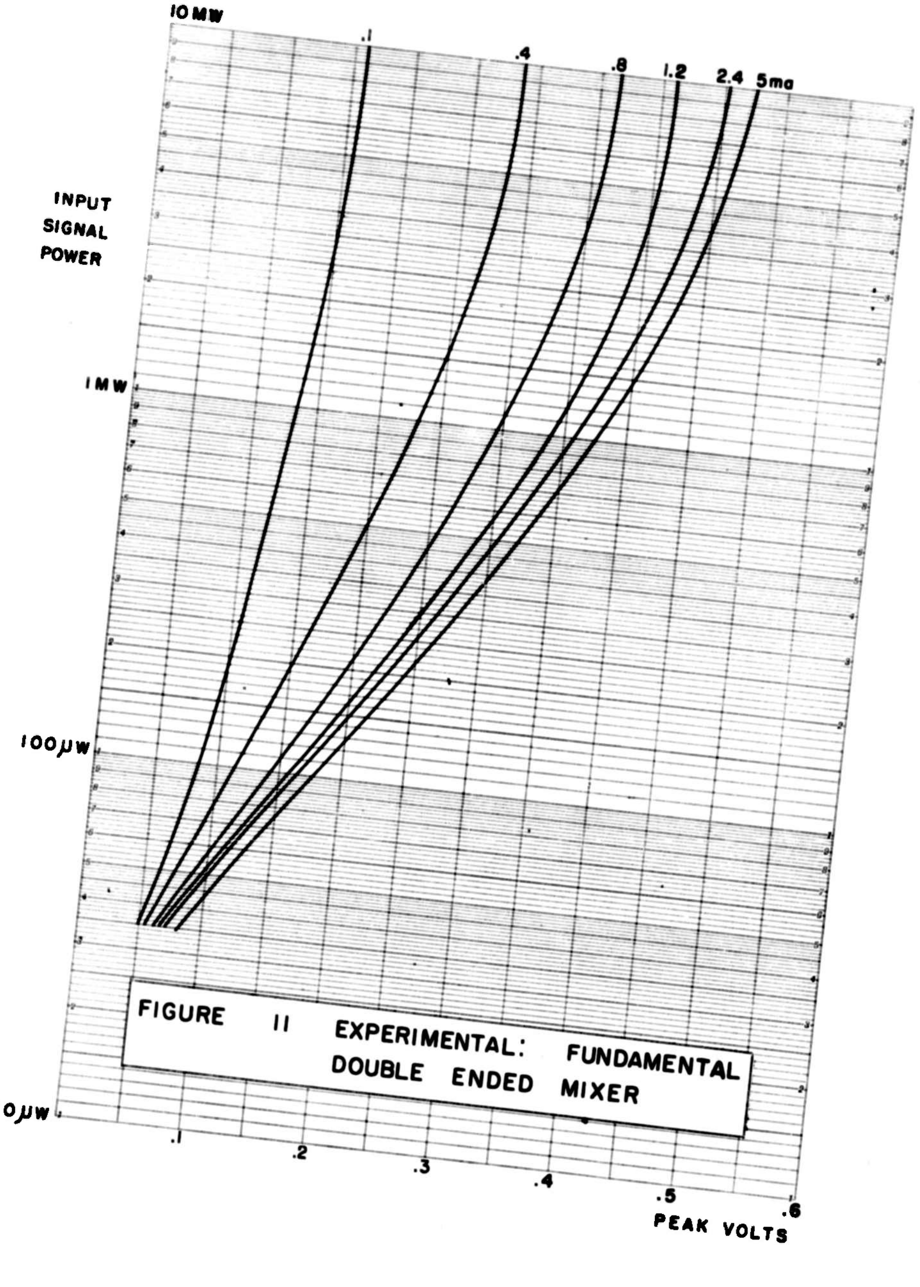

FIGURE 11 EXPERIMENTAL: FUNDAMENTAL DOUBLE ENDED MIXER

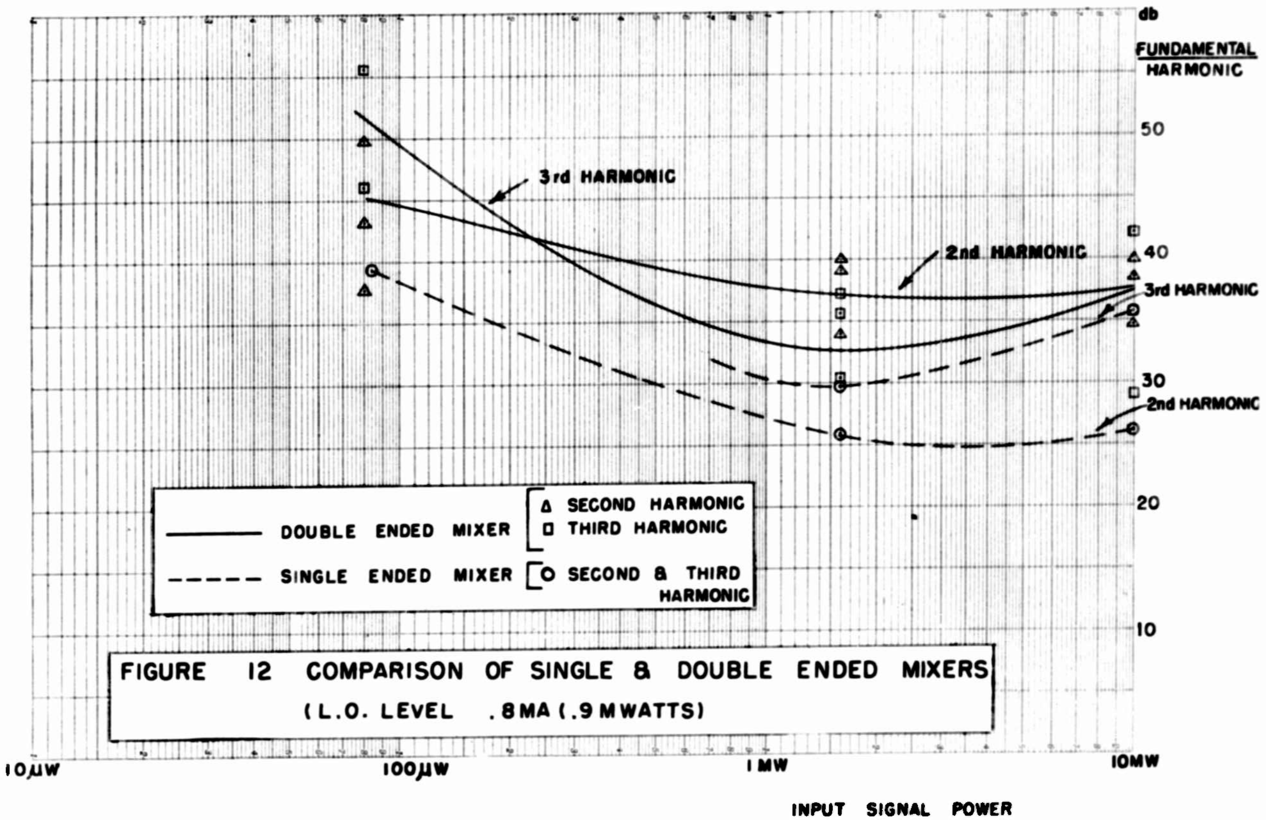

FIGURE 12 COMPARISON OF SINGLE & DOUBLE ENDED MIXERS
(L.O. LEVEL .8MA (.9 MWATTS)

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